

Note on the Influence of the Plate Constants on the Accuracy of the Position of an Object measured on a Photograph. By H. C. Plummer, M.A.

When a preliminary solution is made for the constants of a plate on which the corrected positions of all the stars are required, as in the case of plates for the Astrographic Chart catalogue, the manner of proceeding is sufficiently obvious. It is then simply necessary to employ as large a number of comparison stars as possible, and to use the most accurate positions which can be found for them. The case is, however, rather different when the object is to find, not a solution which is to be applied over the whole extent of the plate, but one which is to be used to correct the measured coordinates of a particular object. This is what is required in parallax work, for example. Here questions arise which concern the choice and number of the comparison stars, and it is important from the point of view of economy to decide the relative degree of accuracy with which their coordinates should be measured. Mr. Filon has discussed the matter very thoroughly* and his main inferences are probably well known. The subject seems sufficiently important to justify discussion in a slightly different way, even if little that is new be added.

The equations from which the plate constants are calculated are of the form

$$ax + by + c = \Delta x ; dx + ey + f = \Delta y$$

where Δx , Δy are the differences between the computed coordinates and the measured coordinates of the comparison stars, *i.e.* the projections on the axes of the distances Δr between the computed and the measured positions. It is assumed that the comparisons are of equal weight for both coordinates and for all the stars. The six-constant solution will be considered first. The normal equations are

$$\begin{aligned} a[x^2] + b[xy] + c[x] &= [x\Delta x] ; d[x^2] + e[xy] + f[x] = [x\Delta y] \\ a[xy] + b[y^2] + c[y] &= [y\Delta x] ; d[xy] + e[y^2] + f[y] = [y\Delta y] \\ a[x] + b[y] + cn &= [\Delta x] ; d[x] + e[y] + fn = [\Delta y] \end{aligned}$$

where n is the number of comparison stars. But it is true, and formal proof might be given if necessary, that had the comparison been made according to any other axes the results would have been equivalent. Hence for theoretical purposes we may take as axes the principal axes of the momental ellipse at the centroid of particles of unit mass occupying the positions of the comparison

* *Monthly Notices*, vol. lxii. p. 561.

stars. The equations for the plate constants then take the simple form

$$\begin{aligned} a[x^2] &= [x\Delta x]; & d[x^2] &= [x\Delta y] \\ b[y^2] &= [y\Delta x]; & e[y^2] &= [y\Delta y] \\ cn &= [\Delta x]; & fn &= [\Delta y]. \end{aligned}$$

If p is the probable error of a difference Δx or Δy , and p_a is the probable error of a , &c., then

$$\begin{aligned} p_a^2[x^2] &= p_b^2[y^2] = p_c^2n \\ &= p_d^2[x^2] = p_e^2[y^2] = p_f^2n = p^2 \end{aligned}$$

Now if x_o, y_o are the coordinates of the object whose position is required; $\delta x_o, \delta y_o$ the errors in x_o, y_o due to errors δa in a , &c.; and p_x, p_y the probable errors of these coordinates so far as the effect of erroneous plate-constants is concerned,

$$\delta x_o = x_o \delta a + y_o \delta b + \delta c; \quad \delta y_o = x_o \delta d + y_o \delta e + \delta f.$$

Hence,

$$\begin{aligned} \delta x_o &= \Sigma \left\{ \delta \Delta x \left(\frac{x_o x}{[x^2]} + \frac{y_o y}{[y^2]} + \frac{1}{n} \right) \right\} \\ \therefore p_x^2 &= p^2 \Sigma \left(\frac{x_o x}{[x^2]} + \frac{y_o y}{[y^2]} + \frac{1}{n} \right)^2 \\ &= p^2 \Sigma \left(\frac{x_o^2 x^2}{[x^2]^2} + \frac{y_o^2 y^2}{[y^2]^2} + \frac{1}{n^2} \right) \\ &= p^2 \left(\frac{x_o^2}{[x^2]} + \frac{y_o^2}{[y^2]} + \frac{1}{n} \right) \dots \dots \dots (1) \\ &= p_y^2, \text{ similarly.} \end{aligned}$$

Again, if the star places be corrected for refraction and aberration, we have

$$e = a, \quad d = -b.$$

Then a four-constant solution is possible and the normal equations are

$$\begin{aligned} a[x^2 + y^2] &= [x\Delta x + y\Delta y]; & b[x^2 + y^2] &= [y\Delta x - x\Delta y] \\ cn &= [\Delta x]; & fn &= [\Delta y] \end{aligned}$$

Hence

$$p_a^2[x^2 + y^2] = p_b^2[x^2 + y^2] = p_c^2n = p_f^2n = p^2$$

In this case

$$\begin{aligned} \delta x_o &= \Sigma \left\{ \frac{x_o(x\delta\Delta x + y\delta\Delta y)}{[x^2 + y^2]} + \frac{y_o(y\delta\Delta x - x\delta\Delta y)}{[x^2 + y^2]} + \frac{\delta\Delta x}{n} \right\} \\ \therefore p_x^2 &= p^2 \Sigma \left\{ \frac{x_o x + y_o y}{[x^2 + y^2]} + \frac{1}{n} \right\}^2 + p^2 \Sigma \left\{ \frac{x_o y - y_o x}{[x^2 + y^2]} \right\}^2 \\ &= p^2 \Sigma \left\{ \frac{x_o^2 x^2 + y_o^2 y^2}{[x^2 + y^2]^2} + \frac{1}{n^2} \right\} + p^2 \Sigma \left\{ \frac{x_o^2 y^2 + y_o^2 x^2}{[x^2 + y^2]^2} \right\} \\ &= p^2 \left(\frac{x_o^2 + y_o^2}{[x^2 + y^2]} + \frac{1}{n} \right) \dots \dots \dots (2) \\ &= p_y^2, \text{ similarly.} \end{aligned}$$

The formulæ (1) and (2) have been obtained without making any assumption as to the configuration of the comparison stars. Moreover, by basing the calculation directly on the errors of the coordinates of the comparison stars, instead of using the probable errors of the plate constants, the necessity of assuming an absence of correlation between these constants has been avoided.

The most important inference is that the object required should be at, or very near, the origin—i.e. should coincide as closely as possible with the centroid of the comparison stars. If this condition be satisfied, the configuration of the stars and the method of reduction (by four or six constants) are practically immaterial. The probable errors, so far as they are due to erroneous constants, are given by

$$p_x = p_y = p / \sqrt{n}$$

If the object does not coincide with the centroid of the comparison stars it is desirable to make the moments of inertia about both axes in the plate large—that is, to have the stars widely scattered in both directions—if the six-constant solution is used; but if the four-constant solution is used it is only necessary that one of the principal moments should be large. The four-constant solution is more valuable when the solution is to apply to the whole extent of the plate, particularly when the distribution of the comparison stars is bad.*

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Note on the formulæ connecting "Standard Coordinates" with Right Ascension and Declination. By F. W. Dyson, M.A., F.R.S.

The simplest and most interesting formulæ from a mathematical point of view for determining Standard Coordinates are those given by M. Trépied in the Introduction to the Algiers Section of the *Astrographic Catalogue* (p. v), and all others are readily deduced from them.

* Mr. Filon, to whom I have had the advantage of showing the above note in MS., points out the desirability of stating definitely the assumptions which underlie the argument—namely, that "there is no correlation between the errors in the residuals of two given stars," and also that "there is no correlation between errors of measurement in the two coordinates" for each star. He adds that "distortion or ellipticity of the images may correlate the errors in x, y ; and, further, that if this distortion affect similarly all the star discs, it may introduce correlation into the x -measures of different stars." But in general, in work of the kind to which this note is intended to apply, the distortion of the images can be considered so slight that no serious effect of this nature need be apprehended.